

DESIGN AUDITS OF MACHINERY: ROTOR DYNAMICS

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Turbomachinery's overall reliability depends heavily on its underlying rotordynamic characteristics. This tutorial briefly reviews some of the important rotordynamic principles and phenomena. Case history examples are presented illustrating the influence of operating conditions on machinery vibration performance.

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1 Introduction

The costs associated with critical machinery downtime in ethylene and other petrochemical plants continue to increase. While machinery OEMs may guarantee their hardware, they do not guarantee these downtime losses in the event the machine does not work reliably over the expected life. Therefore, the end-user bears substantially greater costs and is the one most concerned with minimizing risks. To mitigate the risks of such machinery downtime, there are many types of machinery design audits that can be performed: aerodynamic and thermodynamic performance, blade and impeller stresses, auxiliary system sizing, coupling sizing and rotordynamic performance [2].

The fundamental objective of all such audits is to identify problems before manufacturing to avoid delays during shop testing and downtime in the field. If a rotordynamic problem is encountered after manufacturing begins, the end-user may have to deal with delays in machine delivery. If the problem is not identified until after field commissioning, the implications include reduced production due to restricted operating range (speed, pressure, power, flow, etc.), poor long term reliability/performance reducing time between outages, or even failure threatening life and environment.

Given these implications, it is important to perform rotordynamics audits for not only new machines but also for any modification of existing machines in critical applications. Debottlenecking projects that may involve gas composition or speed changes can adversely affect a machine’s rotordynamics just as much as a bearing, seal or coupling design upgrade.

Fluid dynamics determine how a particular fluid will move through and over surfaces. A machine’s rotordynamics determine how the rotating shaft will move or vibrate as it spins and works with the process fluid. For petrochemical turbomachinery, the vibrations of primary concern are the shaft’s lateral and torsional motions. As shown in Figure 1, lateral vibrations involve the radial or whirling movement of the shaft. Large lateral vibrations can threaten stationary components with tight clearances such as the bearings and seals. Torsional vibration results in twisting of the shaft, where repeated stresses can cause fatigue failure of the shaft.

This paper will focus primarily on rotordynamics involving lateral vibrations. After discussing a few of the key vibration principles and phenomena, the aspects of rotordynamics analysis will be reviewed with specific considerations involving some ethylene plant machinery. A more comprehensive and detailed tutorial on rotordynamics is provided by the American Petroleum Institute (API) [1].

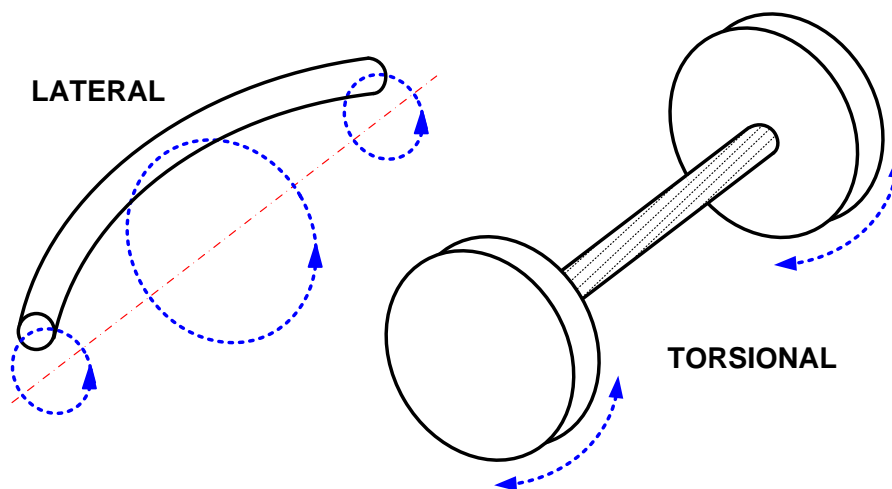


Figure 1: Lateral versus torsional shaft vibrations

2 Vibration Principles and Phenomena

Throughout a machine’s operating range and life (between turnarounds), maintaining small rotor vibration amplitudes is our basic desire. Small rotor vibrations result in low stresses on the shaft, bearings and surrounding equipment. Furthermore, when lateral vibrations are kept at low levels, machine designers are able to implement tighter seal clearances yielding better thermodynamic efficiency of the machine. Though numerous vibration phenomena threaten these benefits, two deserve the most attention, resonance and instability.

2.1 Resonance

Resonances are a concern because small excitations can be amplified to yield very large vibrations. Strictly speaking, a resonance condition occurs when an excitation’s frequency nearly coincides with a system natural frequency. An opera singer breaking a wine glass is a classic example of resonance, where the singer’s pitch nearly equals the natural frequency of the glass. Even though the force of the sound waves is very small, their frequency causes large vibrations in the glass that eventually cause it to crack.

Just like wine glasses, guitar strings and tuning forks, shafts and machine structures have frequencies where they naturally prefer to oscillate when excited. These natural frequencies are dependent on various properties of the system. For a simple weight hung from a spring, its natural frequency, ω_n , equals:

$$\omega_n = \sqrt{\frac{k}{m}}$$

where k is the spring’s stiffness and m is the weight’s mass. A turbomachine will have many natural frequencies, not just one, that are also dependent on its stiffness and mass properties.

As mentioned earlier, the recipe for resonance demands not only a natural frequency, but also the presence of an excitation source with a similar frequency. The biggest excitation source of concern within turbomachinery is shaft unbalance. Just like an unbalanced washing machine load or an unbalanced tire, shaft unbalance occurs when the center of mass is not coincident with the center of rotation. Caused by material imperfections within the shaft, manufacturing tolerances and anything that affects the rotational mass distribution (such as fouling), unbalance can never be totally eliminated.

The excitation force produced by unbalance has the same frequency as the rotor speed. Therefore, any natural frequency from zero to trip speed can be excited by shaft unbalance. As shown in Figure 2, whenever the rotor speed coincides with a natural frequency, the lateral vibration amplitudes can increase dramatically. These resonance situations are called “critical speeds.”

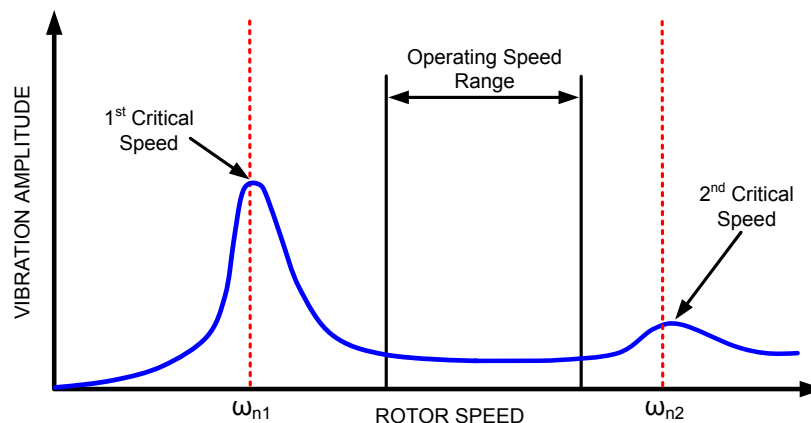


Figure 2: Bode plot of vibrations versus rotor speed

Whether or not a critical speed will cause large vibrations is highly dependent on damping in the system. Adding wine to the glass makes it more difficult, if not impossible, for the singer to crack the glass. Relative to the glass material by itself, the wine has very large damping helping to decay any vibrations quickly. The oil film within the journal bearings provides the main source of damping for many turbomachines. However, for pumps, the main source of damping comes from the process fluid within the close clearance wear rings.

Figure 3 shows how increasing the bearings' damping can reduce vibrations when going through a critical speed. However, bearings have a limit to how much damping they can provide and this may not be enough to restrict critical speed vibration levels. When this is the case, the critical speed must be avoided through startup procedures (quickly accelerating through it) as well as by ensuring adequate separation margin with the operating speed range. As shown in Figure 2, the first critical speed often has less damping than the second critical speed. Most large compressors and turbines are designed to operate between their first and second critical speeds.

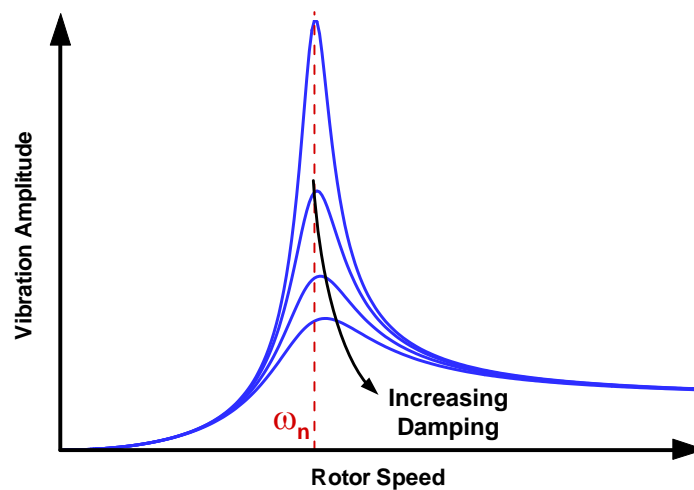


Figure 3: Bearing damping's influence on critical speed amplification

2.2 Instability

Resonances and critical speeds are phenomena falling into the class of forced vibration problems. Without the presence of an excitation source like unbalance, the vibration would not occur. Another class of vibration problems exists, however, which are self-induced or self-excited in nature. Self-excited implies that the forces causing the vibration are controlled by the vibration itself. Turbomachines can experience these self-excited vibrations through a phenomenon called rotor instability [4].

A common example of self-excited instability is the unpleasant squeal sometimes heard to due feedback within a microphone-amplifier-speaker system. The frequency of the squeal is actually the natural frequency of the sound system, and it is so loud because the system has no damping allowing the squeal to grown uncontrollably. Eliminating the squeal is accomplished by reducing the amount of feedback by either turning down the volume, moving the speakers, or using a directional microphone.

Acting just like feedback mechanisms, various components within a turbomachine produce forces whose magnitude and direction are functions of the rotor's motion. Their strength is often dependent on rotor speed and the process fluid's pressure and density. Figure 4 shows how these destabilizing mechanisms encourage vibration by acting tangential to the rotor's whirling motion. Battling against these destabilizing forces is damping. When the damping forces are larger than the destabilizing forces, the vibrations remain stable.

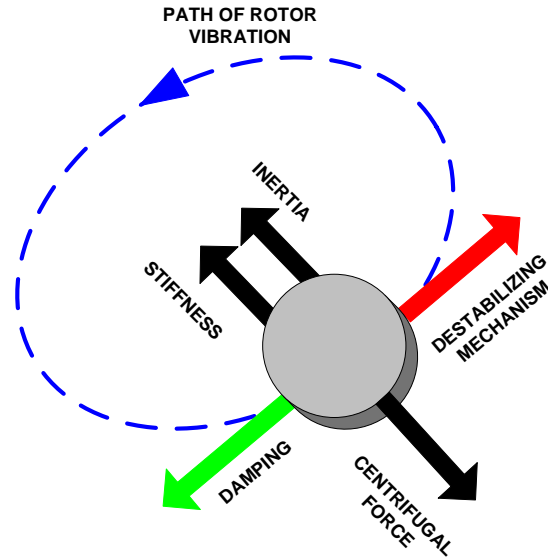


Figure 4: Forces battling over rotor stability

Figure 5 shows how rotor vibrations can grow when destabilizing forces win the battle. For this experiment on an actual rotor, the destabilizing mechanism could be manually turned on or off. When turned on, the rotor vibrations grew uncontrollably to very large levels until 3.2 seconds when the mechanism was turned off and stable vibration returned. Unfortunately, it's difficult to simply turn off these destabilizing mechanisms on industrial turbomachines.

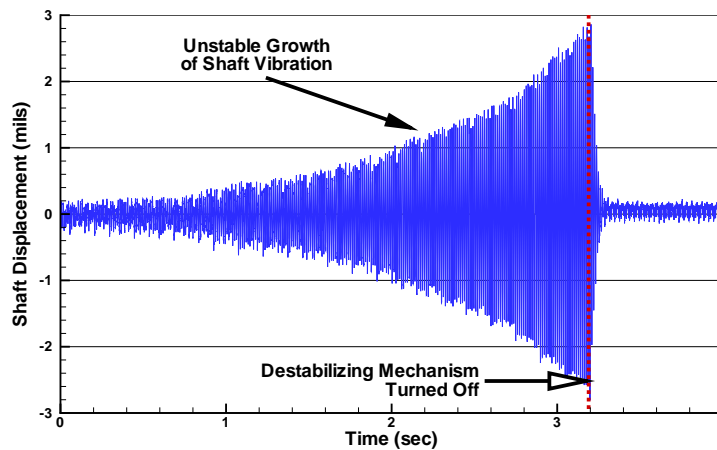


Figure 5: Vibrations of an unstable rotor

Unstable vibrations are able to reach such large levels because of the immense energy available within the spinning rotor. Like a flywheel which stores energy, the spinning motion of a turbomachine's rotor creates a large kinetic energy reservoir. The whirling vibration of the shaft is a separate reservoir that is hopefully small. Destabilizing mechanisms provide a pipeline between these two energy sources, stealing rotational energy and pumping it into whirling vibration energy [3]. When the energy injected into the whirling vibration exceeds the energy dissipated by damping, the rotor vibrations become unstable and large because their source, rotational energy, is so large.

Another reason rotor instability can lead to large vibrations is because the rotor's natural frequency is excited. Figure 6 shows the typical situation when a compressor experienced instability. Each line represents the frequency spectrum at a particular rotor speed. At 6,500 rpm, the only vibration present was due to unbalance with a frequency component at 108 Hz (equal to 6500 cpm). As speed increased, however, destabilizing forces began winning the battle over damping causing the rotor to also vibrate at its natural frequency near 70 Hz. When the rotor speed reached 9,500 rpm, the rotor instability generated very large vibrations at the natural frequency. Unable to reach its maximum continuous speed of 10,000 rpm, operation of the compressor was limited to 9,000 rpm to avoid the damaging nature of this instability.

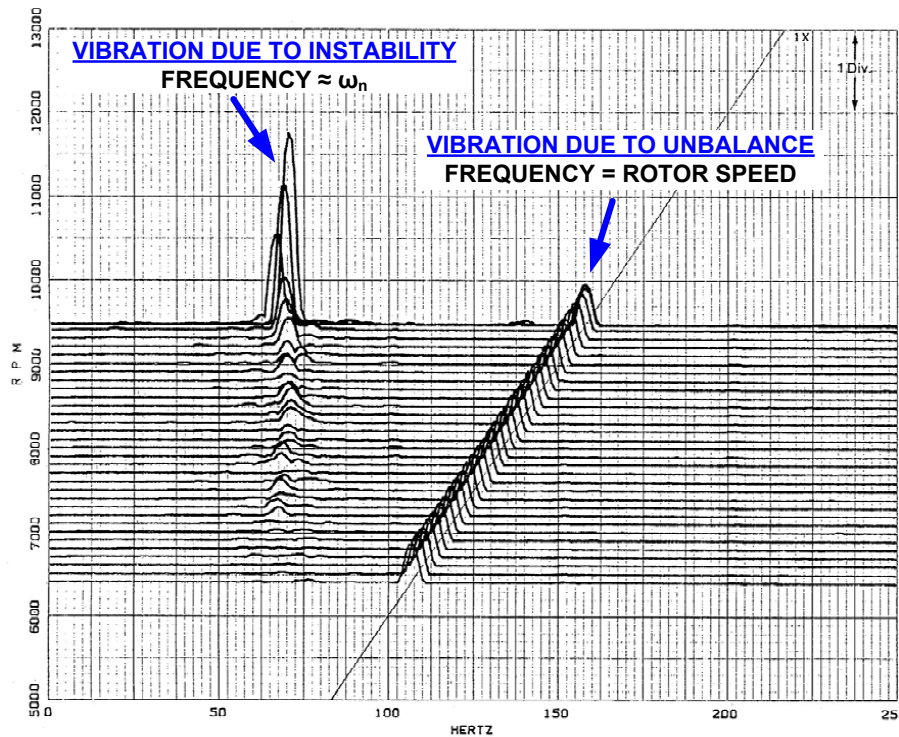


Figure 6: Vibrations due to rotor instability versus unbalance

Restriction of the operating speed range is just one of the costly implications possible with resonance and instability vibration problems. Other restrictions may be imposed such as limiting the machine's inlet flow or the discharge pressure. The vibrations, in some cases, may be such that the machine is essentially inoperable. In other extreme cases, the machine can be completely destroyed before there is even time to shut it down. Developing an accurate model of the rotor's dynamics allows the design to be audited, helping to avoid these collateral problems.

3 Rotordynamic Modeling and Auditing Aspects

Predicting or simulating the vibrations of a turbomachine involves transforming the physical system into a mathematical representation. This is accomplished by modeling all the various components within the machine (rotor, bearings, seals, coupling, etc.) as a combination of masses, springs or dampers. Various engineering disciplines are necessary to develop these component models. Solid mechanics principles are applied to transform the shaft into a series of masses interconnected by springs. Modeling the stiffness and damping properties of the fluids within the bearings and seals relies heavily on fluid dynamic and heat transfer principles [6]. Once all the components are modeled, their masses, springs and dampers are assembled together into a rotordynamic system like that shown in Figure 7.

To develop this rotordynamic model requires extensive design information from the machinery manufacturer. Maintaining the highly proprietary and confidential nature of this information is essential for the end-user or independent party conducting the audit. For the rotor, this information includes detailed dimensions across the entire length as well as the weights and locations of all mounted components such as impellers and couplings. Journal bearing design information needs include dimensions, clearances, as well as material and lubricant properties. API provides a detailed listing of the necessary information.

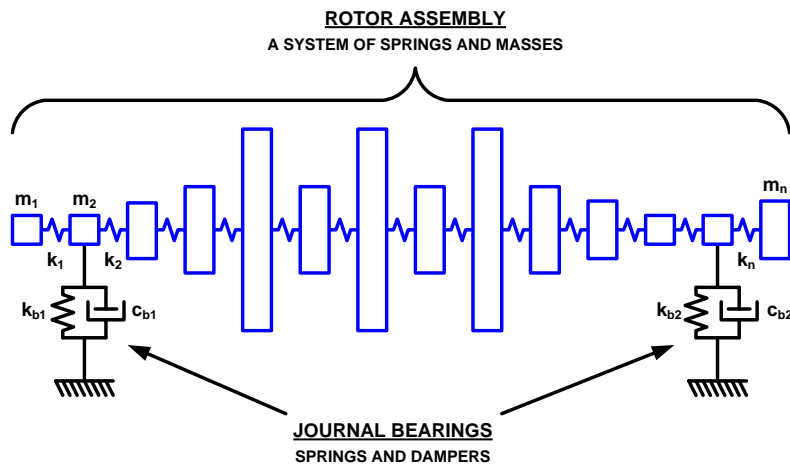


Figure 7: Dynamic model of rotor system

Once the rotordynamic model is completed, it is used in various analyses to predict how the machine will vibrate during operation. The main lateral rotordynamic analyses required by API are unbalance response and stability. These two analyses address the two largest vibration threats that were highlighted earlier, resonance and instability.

3.1 Unbalance Response Analysis

Unbalance response analysis involves applying known unbalance(s) to the rotordynamic model and simulating startup/shutdown of the machine. This analysis predicts whether or not the following design objectives will be achieved:

- Avoid resonance situations with respect to the operating speed range (maintain an adequate separation margin away from lightly damped critical speeds),
- Maintain low vibrations (much smaller than the smallest clearance) in the operating range that are robust to balance degradation over time (such as due to fouling, up to a certain practical limit),

- Ensure vibrations do not cause damage during startup and trip (no reduction in efficiency/performance due to rubbing/wear of seals).

Figure 8 presents the results of an unbalance response analysis of a 40,000 hp (30 MW) propylene refrigeration compressor. The first critical speed occurs at 1,545 rpm with a large vibration peak. Within the operating speed range, the vibration is much lower than the API limit of 0.001 inch (25.4 μm) p-p which is very desirable for long term operation. No resonance problems are present, since both the first and second critical speeds are well separated from the operating speed range. API requires a certain amount of separation margin depending on the critical speed's level of amplification (the peak's sharpness).

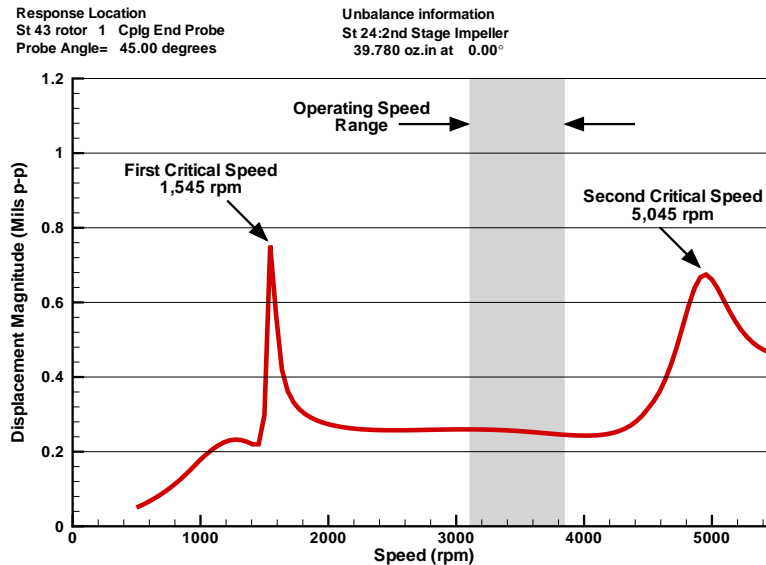


Figure 8: Propylene refrigeration compressor unbalance response

Manufacturing tolerances exist within the journal bearings, and oil supply conditions may change. These potential variations must be also examined to ensure the vibration design objectives are maintained.

Even though the propylene refrigeration compressor's vibrations are low in the operating speed range of Figure 8, they become much larger when going through the first critical speed. During startup, these vibrations at the first critical speed will be avoided by quickly accelerating through this speed. However, during a coastdown, the deceleration rate may be very slow allowing these vibration levels to threaten the tight clearances within the machine.

A rub check analysis is conducted to assess whether or not rubbing will occur. Figure 9 presents the deflected shape of the rotor at the first critical speed. It is important to notice that vibrations at the five impellers within the machine are much larger than those near the bearings, where vibration measurements are typically acquired. These vibrations at the impellers must be compared against the impellers' seal clearances to make sure that rotor-seal contact will not occur, damaging the seals and reducing compressor performance. Many compressors are designed with abradable seals that allow for some limited contact/wear without loss of performance.

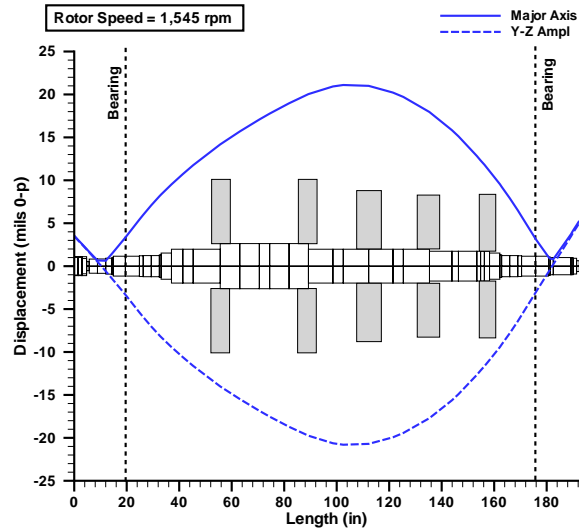


Figure 9: Vibration levels along propylene refrigeration compressor's rotor at first critical speed

3.2 Stability Analysis

To avoid the rotor instability phenomenon discussed earlier, stability analysis becomes another vital part of the rotordynamic audit process. Unlike the unbalance response analysis which is conducted across the entire speed range, stability analysis is typically only conducted at maximum continuous speed. Two levels of stability analysis are conducted, where the first consists of a screening analysis to assess:

- Stability level with an empirical estimate of the destabilizing forces present,
- Stability robustness, and
- Stability relative to industrial experience (centrifugal compressors only).

If the screening analysis reveals a marginal machine, a second analysis level is required involving state-of-the-art, detailed modeling of seals and other potentially destabilizing components. API provides specific screening and acceptance criteria for both of these levels of analysis [1].

The stability sensitivity curve is one of the main products of the stability screening analysis. Figure 10 presents this curve for the propylene refrigeration compressor discussed earlier. As the level of destabilizing force is increased, stability decreases until it reaches the threshold and unstable vibrations would occur. Because of the uncertainty in the actual bearing condition, this sensitivity analysis is conducted for the extremes of the bearing tolerances. Figure 10 shows that, at minimum clearance bearing conditions, the compressor is less robust to destabilizing forces than under maximum clearance conditions.

The estimated level of destabilizing force in Figure 10 is determined based on the speed as well as suction and discharge gas densities at rated compressor operating conditions. Because operating conditions can change, it is important that the machine be robust enough to resist the resulting variations in destabilizing forces. Figure 10 indicates this propylene refrigeration compressor will not be able to handle a large increase in destabilizing forces if the bearings are at their minimum clearance.

3.3 Specific Concerns with Ethylene Plant Machinery

During the audit of critical ethylene plant machinery, there are several rotordynamic concerns that typically arise. The rotors within modern cracked gas and refrigeration trains tend to be very large and heavy. This causes their first critical speeds to be very amplified with large vibrations, threatening bearing and

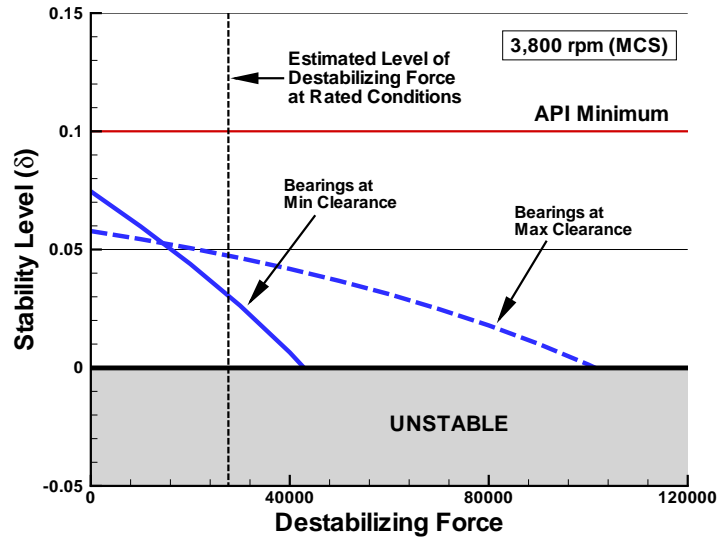


Figure 10: Stability sensitivity curve for the propylene refrigeration compressor

seal clearances during startup and shutdown. These heavy rotors also often lead to the machines' supporting pedestals becoming important dynamically. Therefore, rotordynamic analyses need to consider these pedestal dynamics in addition to those of the shaft and bearings [10].

Today, many cryogenic turboexpanders are being designed with active magnetic bearings in place of fluid film bearings. Active magnetic bearings have many attractive mechanical and operational features, but also add significant complexity to the rotordynamic auditing process. The rotor, magnetic bearings, as well as their electronic controls must all be modeled and combined into a single system for the dynamic analyses [9].

3.4 Shop Testing versus Analytical Auditing

Some may argue that rotordynamic auditing is not necessary since their machine will be tested in the manufacturer's shop before shipment. Shop testing is certainly an important measure for avoiding field vibration problems. Such testing usually consists of the following:

- A mechanical run test where the machine achieves full speed, but under essentially no load conditions,
- A performance test using an inert gas mixture at reduced speeds and pressures.

Mechanical run tests are useful for identifying critical speed or resonance problems. Unfortunately, neither type of test does a good job of uncovering stability problems because the level of destabilizing forces is not present. An alternative performance test at full speed, power and gas density can be conducted exposing the machine to realistic levels destabilizing forces. However, the high costs associated with this alternative type of performance testing often preclude it from many projects.

Regardless of the shop testing conducted, field conditions can never be totally replicated in the manufacturer's shop. This reality, combined with the fact that any problem identified during shop testing will likely cause a delay in the machine's delivery, makes auditing during the design process a worthwhile initiative. Furthermore, rotordynamic auditing provides the flexibility to examine the machine's robustness to various situations that are likely in field operation, but may not be practical for shop testing. These include variations in gas molecular weight, process fluid density, bearing oil supply temperature, as well as additional unbalance due to fouling.

4 Case Histories

To demonstrate some of these rotordynamic principles, several case histories will now be examined. Two of these cases illustrate how rotordynamic auditing during the design phase could have easily avoided subsequent vibration problems.

4.1 Cryogenic Turboexpander

In 1939, Kapitza [7, 8] introduced a radial inflow, expansion turbine for cryogenic air separation, a design that would become the father of modern turboexpanders used in ethylene plants. Figure 11 presents a cross section of the Nobel laureate's turboexpander. During the development of this turboexpander, Kapitza encountered what is likely the first documented case of a turbomachine's process fluid (in this case, air) acting like a destabilizing mechanism and causing rotor instability.

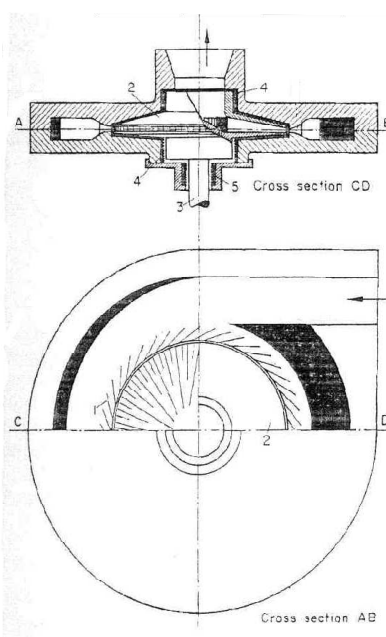


Figure 11: Kapitza's cryogenic expansion turbine [7]

To achieve the high thermal efficiency desired, the turboexpander's seal clearances had to be very small (approximately 0.006 inch (0.15 mm)) between the stator and rotor spinning at 40,000-50,000 rpm. Therefore, the rotor's vibrations had to be kept sufficiently small in order to avoid mechanical contact with these tight seal clearances.

With room temperature air supplied to the turboexpander, no vibration problems were encountered. However, at the low air temperatures required for air liquefaction, the turboexpander could not operate at full speed because of large rotor vibrations damaging the seals. Figure 12 illustrates the type of shaft vibrations observed by Kapitza. The shaft's orbital motion grew with each cycle eventually making contact with the stationary seals. The small, higher frequency oscillations present in Figure 12 are due to unbalance.

Kapitza correctly identified the cause of this instability to be the increased air density, which was three to five times higher than that at room temperature. Recognizing the need for damping to win the battle against air's destabilizing force (see Figure 4), Kapitza implemented a squeeze film damper that eliminated the instability. The turboexpander was then able to achieve normal operating speed and efficiency because its seals were not being damaged. Such squeeze film dampers are commonly employed in modern jet aircraft engines.

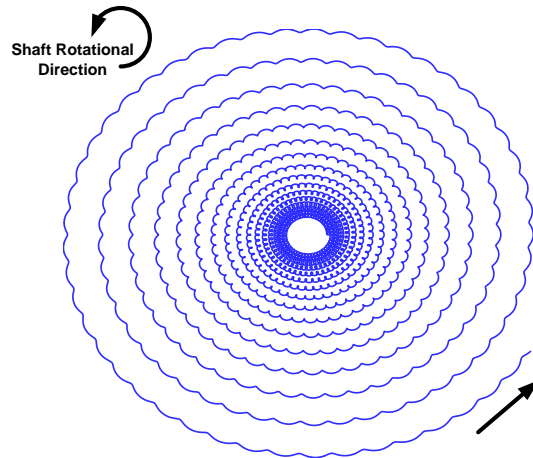


Figure 12: Unstable orbital shaft motion observed by Kapitza

Kapitza’s turboexpander provides a good example of the implications when a rotordynamic vibration problem is encountered. Until a time-intensive design solution could be implemented, the machine’s operation was limited and his development project delayed. The stakes are raised, however, when a vibration problem occurs on a machine critical to a modern petrochemical plant. Such was the case on a rerated cracked gas compressor.

4.2 Cracked Gas Compressor

During commissioning of the rerated 11-stage cracked gas compressor, a surge event occurred causing large shaft vibrations that wrecked the machine before it could be shutdown. Figure 13 presents the orbital motion at one of the compressor’s bearings an instant before failure. The amplitude of vibration was over twelve times the normal level.

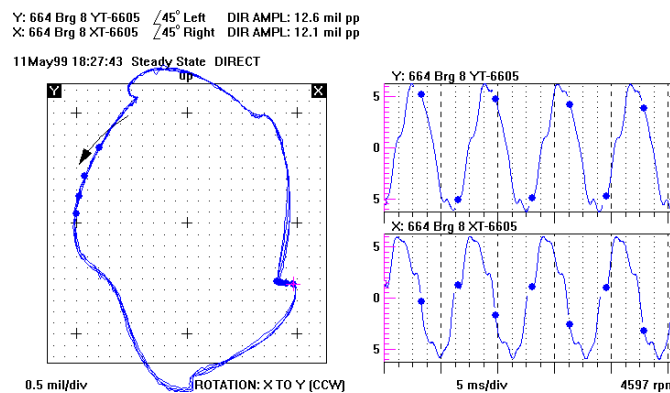


Figure 13: Large shaft vibrations on cracked gas compressor

Using spare parts, the compressor was rebuilt and improvements were made to the surge control system. However, upon restart, the compressor surged wrecking the entire compressor once again. While not desirable in centrifugal compressors, surge does not typically lead to large radial or lateral vibrations of the shaft like those experienced here. Surge, in this case, acted as a catalyst or trigger for the underlying cause of the high vibrations, rotor instability.

Post-mortem rotordynamic analyses by the manufacturer and end-user confirmed that the machine was prone to instability problems. Like Kapitza, more damping in the bearing system was required and

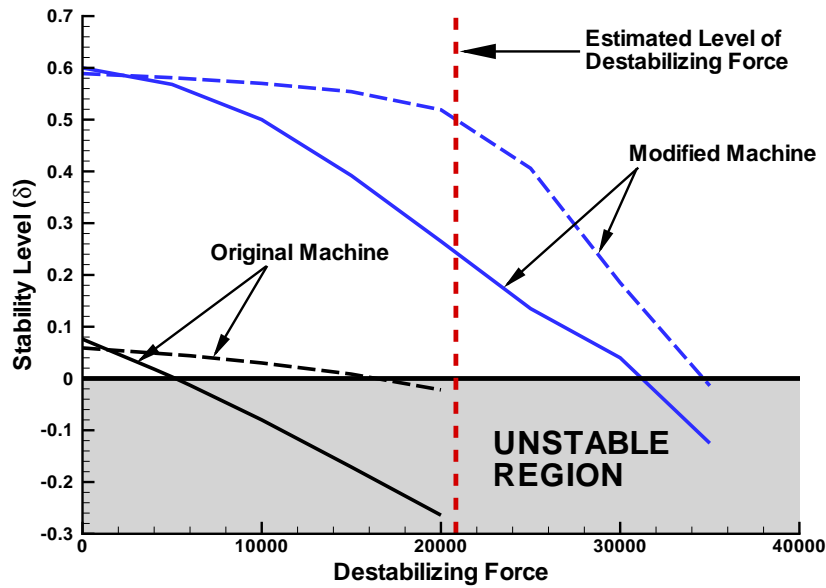


Figure 14: Stability sensitivity curves for original and modified cracked gas compressor at minimum (solid line) and maximum (dashed) bearing clearance conditions

a squeeze film damper design was implemented. Figure 14 presents the resulting improvement in the machine's stability characteristics.

Until the machine's design could be modified, the entire ethylene plant's startup was delayed by one month due to this vibration problem. The root cause of this problem was eventually attributed to a simple human error in the manufacturer's original stability analysis. Furthermore, no design audit had been performed by the end-user. If such an audit had been performed, it would have likely avoided the millions of dollars in production losses. A simple benefit-cost ratio calculation for this audit would be on the order 1000 to 1. In other words, every dollar invested in the rotordynamic audit would have avoided \$1000 in production losses.

4.3 Ethylene Product Pump Motor

During shop no-load testing, high vibrations were measured on a vertical 1,200 hp (895 kW) induction motor spinning at 3,600 rpm. This new motor was being purchased to replace the pump's existing motor with a higher power model. No rotordynamic design audit had been conducted by the end-user.

Figure 15 presents the motor's vibrations measured during coastdown. At running speed, vibration levels are shown to exceed 0.2 in/sec (5 mm/sec) 0-peak, well above the specified limit. The manufacturer attributed the peak at approximately 1,200 rpm to be the motor's first critical speed, but could not explain the large increase in vibration as speed was increased.

The manufacturer had conducted no rotordynamic analysis on this motor design. Therefore, models of the motor's stator casing and rotor were developed and coupled into a single dynamic system. Figure 16 presents a schematic of this model. Unbalance response analysis identified the vibration problem to be caused by a critical speed separation margin problem. The motor's running speed and a critical speed at 3,700 rpm were in a state of resonance.

The analysis also identified the 1,200 rpm peak to be associated with a natural frequency where the entire motor housing is pivoting about its bottom mounting flange. Commonly referred to as a "reed critical frequency" [5], this natural frequency is not associated with the rotor's critical speed as thought by the motor manufacturer.

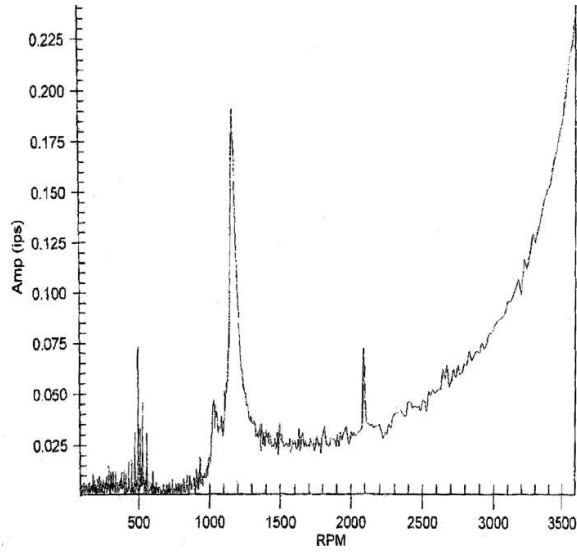


Figure 15: Motor vibration level versus speed

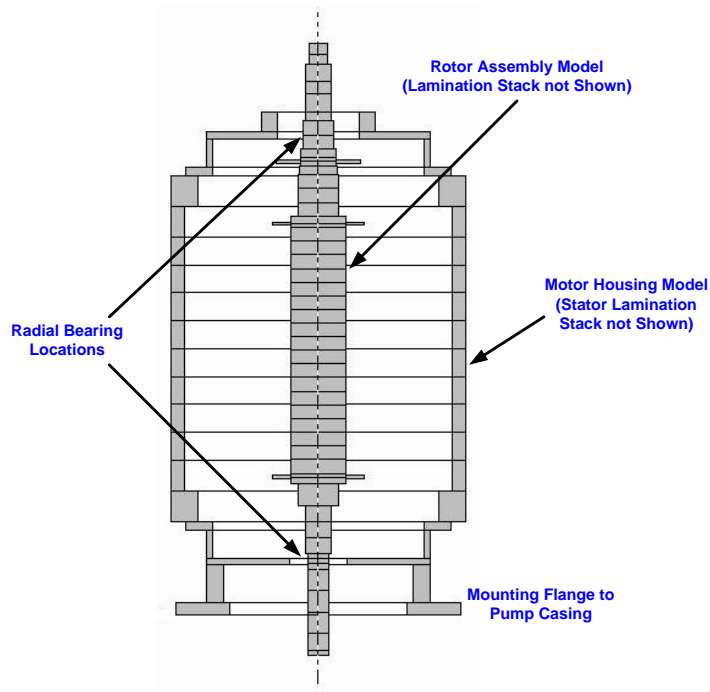


Figure 16: Schematic of motor's casing and rotor models

Major shaft modifications were required to correct the resonance problem with the rotor's critical speed. These modifications delayed motor shipment by three months.

5 Summary and Recommendations

A turbomachine’s underlying rotordynamic characteristics determine how its shaft will vibrate. Two vibration phenomena, resonance and instability, are the greatest threats to the turbomachine being delivered on time, achieving the process’s performance requirements, and running reliably between planned outages. Independent rotordynamic audits during the design phase can help identify these problematic vibration phenomena before they cause shipment delays or downtime in the field.

One of many types of design audits that can be performed, rotordynamic auditing should especially be considered for critical, non-spared machinery as well as for new machine designs without significant experience. Auditing is also highly recommended for machine rerates and any modifications to the rotor assembly, bearings, seals and coupling(s).

Even as modeling and design tools improve, the threat of poor rotordynamic performance will always be a concern. Several realities drive this continued threat. First, continuing improvements and increasing demands of industrial processes require machinery to operate at higher speeds, pressures and power levels. Second, competitive pressures force OEMs to push existing designs beyond their proven envelopes.

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